



First Name

Last Name

## **YT Tutoring Center**

### **Principles of Mathematics 12**

### **Sample Exam 2008 (Version A)**

## GENERAL INSTRUCTIONS

1. Aside from an approved calculator, electronic devices, including dictionaries and pagers, are **not** permitted in the examination room.
2. All multiple-choice answers must be entered on the Response Forms using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
3. For each of the written-response questions, write your answer in the space provided in this booklet. Rough-work space has been incorporated into the space allowed for answering each written-response question. You may not need all of the space provided to answer each question.
4. Ensure that you use language and content appropriate to the purpose and audience of this examination. Failure to comply may result in your paper being awarded a zero.
5. This examination is designed to be completed in **two hours**. *Students may, however, take up to 30 minutes of additional time to finish.*

## PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

	Value	Suggested Time	Allowable Time
1. This examination consists of <b>two</b> parts:			
PART A:			
Section I: 16 multiple-choice questions	24 marks	35 minutes	45 minutes
<b>Note:</b> No calculator may be used for the first 45 minutes of the examination.			
Section II: 28 multiple-choice questions (some of which require the use of a calculator)	42 marks	55 minutes	} 105 minutes
PART B: 5 written-response topics covered by 8 questions	24 marks	30 minutes	
<b>Total: 90 marks    120 minutes    150 minutes</b>			

2. **After 45 minutes, the blue Response Form (Section I) will be collected. When all blue Response Forms are handed in, you will be permitted to use your calculator.**

During the first 45 minutes, you may proceed to other questions on the examination, many of which do not require the use of a calculator. Once the blue Response Forms have been handed in, you will **not** be able to go back to any of the first 16 questions; therefore, ensure you have checked your answers before proceeding to the rest of the examination.

3. The last **four** pages inside the back cover contain **A Summary of Basic Identities and Formulae, The Standard Normal Distribution Table, Rough Work for Graphing,** and **Rough Work for Multiple Choice.**
4. When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

**PART A: MULTIPLE CHOICE (non-calculator)**

**SECTION I**

**Suggested Time: 35 minutes**

**Value: 24 marks**

**Allowable Time: 45 minutes**

**INSTRUCTIONS:** No calculator may be used for this section of the examination. For each question, select the **best** answer and record your choice on the blue Response Form provided. Using an HB pencil, completely fill in the circle on the blue Response Form that has the letter corresponding to your answer.

1. Determine the exact value of  $\csc \frac{7\pi}{4}$

- A.  $-\sqrt{2}$
- B.  $\sqrt{2}$
- C.  $-\frac{1}{\sqrt{2}}$
- D.  $\frac{1}{\sqrt{2}}$

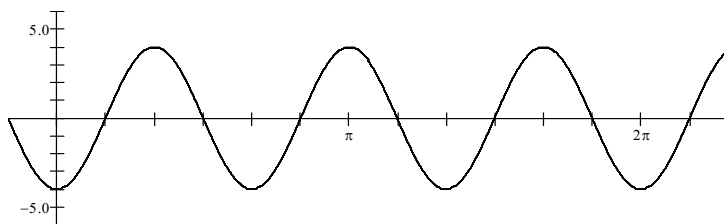
2. Determine the phase shift of the function  $f(x) = \cos\left(\frac{1}{2}x + 3\pi\right)$

- A.  $\frac{2\pi}{3}$  to the left
- B.  $\frac{3\pi}{2}$  to the left
- C.  $3\pi$  to the left
- D.  $6\pi$  to the left

3. Determine the exact value of  $\csc\left(-\frac{7\pi}{6}\right)$

- A. 2
- B. -2
- C.  $\frac{1}{2}$
- D.  $-\frac{1}{2}$

4. Determine an equation of the cosine function graphed below.



- A.  $y = -4 \cos 3x$
- B.  $y = -4 \cos \frac{1}{3}x$
- C.  $y = 4 \cos 3x$
- D.  $y = 4 \cos \frac{1}{3}x$
5. Which of the following is an equation of an asymptote of the function  $y = \sec 4x$ ?
- A.  $x = \frac{\pi}{16}$
- B.  $x = \frac{\pi}{8}$
- C.  $x = \frac{\pi}{4}$
- D.  $x = \frac{\pi}{2}$
6. Solve:  $1 + 2 \sin x = 0$ ,  $0 \leq x < 2\pi$ .
- A.  $-\frac{\pi}{6}$
- B.  $\frac{2\pi}{3}, \frac{5\pi}{3}$
- C.  $\frac{2\pi}{3}, -\frac{\pi}{3}$
- D.  $\frac{7\pi}{6}, \frac{11\pi}{6}$

7. Determine an expression equivalent to  $18\cos^2 10x - 9$

- A.  $9 \cos 5x$
- B.  $9 \cos 20x$
- C.  $-9 \cos 5x$
- D.  $-9 \cos 20x$

8. Determine the domain of the function  $y = \log_3(x+2) - 1$

- A.  $x < -2$
- B.  $x > -2$
- C.  $x < 2$
- D.  $x > 2$

9. Determine an equivalent expression for  $\log\left(\frac{A}{5B}\right)$

- A.  $\frac{\log A}{5\log B}$
- B.  $\frac{\log A}{\log 5 + \log B}$
- C.  $\log A - \log 5 + \log B$
- D.  $\log A - \log 5 - \log B$

10. Simplify  $\log_2 4^x$

- A.  $x$
- B.  $2x$
- C.  $2^x$
- D.  $x^2$

11. In chemistry, the pH scale measures the acidity (0–7) or alkalinity (7–14) of a solution. It is a logarithmic scale in base 10. Thus a pH of 9 is 10 times more alkaline than a pH of 8. If a solution has a pH of 6.9 and a second solution is 1000 times more alkaline, determine the pH of the second solution.

- A. 8.9
- B. 9.9
- C. 4.9
- D. 3.9

12. Alan invests \$10,000 in a bond which pays interest at the rate of 8% per annum compounded quarterly. Which equation can be used to determine the final amount  $A$  of this investment in 5 years?

- A.  $A = 1000(1.02)^5$
- B.  $A = 10000(1.02)^{20}$
- C.  $A = 10000(1.08)^5$
- D.  $A = 10000(1.08)^{20}$

13. Given  $\log_a 2 = x$  and  $(\log_a 8)(a^{\log_a x}) = 12$ , solve for  $x$ .

- A. 2
- B.  $\pm 2$
- C.  $\sqrt{2}$
- D.  $\pm\sqrt{2}$

14. Solve for  $x$ :  $a^{x+3} = b^{x-2}$

A.  $\frac{2 \log b - 3 \log a}{\log b - \log a}$

B.  $\frac{2 \log b - 3 \log a}{\log a - \log b}$

C.  $\frac{2 \log b + 3 \log a}{\log b - \log a}$

D.  $\frac{2 \log b + 3 \log a}{\log a - \log b}$

15. Solve for  $n$ :  ${}_nP_2 = 42$

A. 2

B. 6

C. 7

D. 42

16. Simplify the following expression without using the factorial symbol  $\frac{(n+2)!(n-1)!}{((n+1)!)^2}$ .

A.  $\frac{1}{n+1}$

B.  $\frac{(n+2)}{(n+1)(n)^2}$

C.  $\frac{(n+2)}{(n+1)(n)}$

D.  $\frac{(n-1)}{(n+1)(n)}$



**PART A: MULTIPLE CHOICE**

**SECTION II**

**Value: 42 marks**

**Suggested Time: 55 minutes**

**INSTRUCTIONS:** For each question, select the **best** answer and record your choice on the blue Response Form provided. Using an HB pencil, completely fill in the circle on the blue Response Form that has the letter corresponding to your answer.

17. Determine the inverse of the function  $f(x) = \frac{3x-1}{2x}$

A.  $\frac{2x-1}{3x}$

B.  $\frac{1}{3x-2}$

C.  $\frac{1}{3-2x}$

D.  $\frac{1}{2x-3}$

18. The y-intercept of the function  $y = f(x)$  is  $-1$ . Determine the y-intercept of  $y = -\frac{1}{f(x)} - 1$

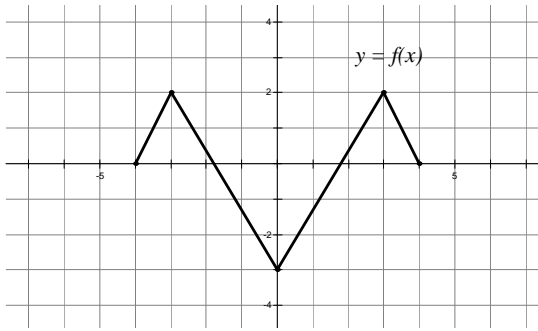
A. 1

B. 0

C. -1

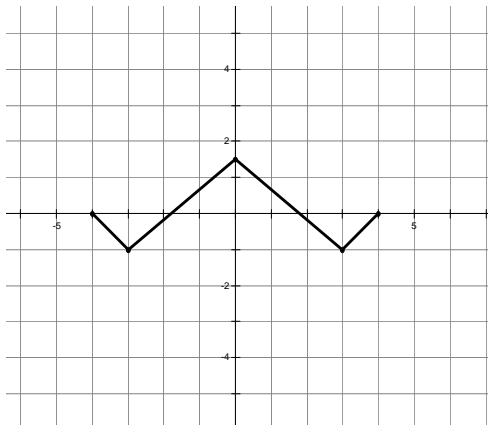
D. -2

19. The graph of the function  $y = f(x)$  is shown below.

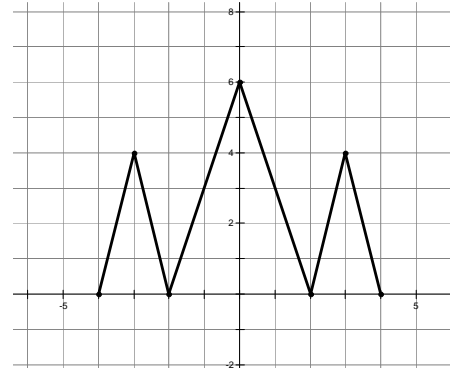


Determine the graph of  $y = 2|f(x)|$

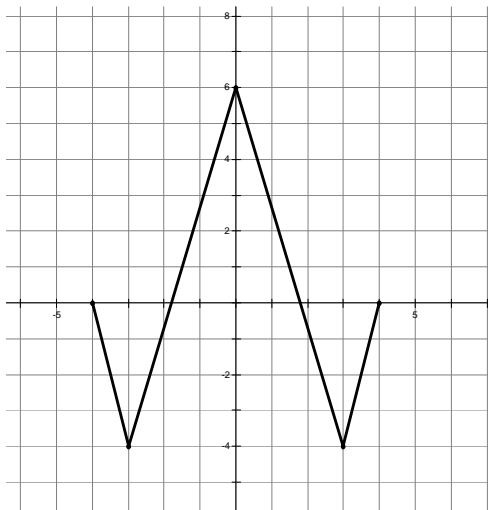
A.



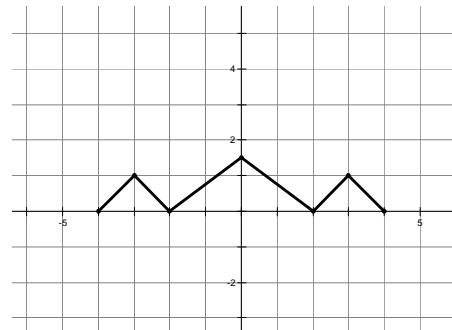
C.



B.



D.

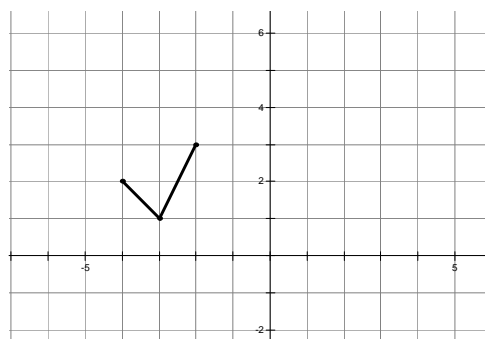
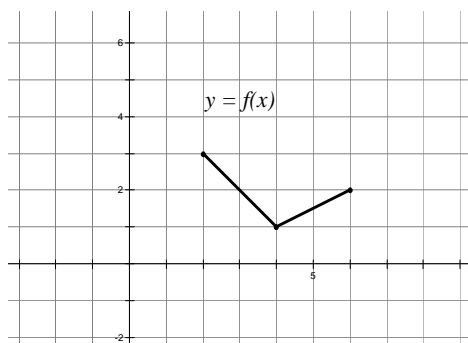


20. If the point  $(-3, -6)$  is on the graph of  $y = f(x)$ , what point must be on the graph of

$$y = -f\left(\frac{1}{3}x - 1\right).$$

- A.  $(2, 6)$
- B.  $(0, 6)$
- C.  $(-8, 6)$
- D.  $(-6, 6)$

21. The graph of  $y = f(x)$  is shown on the left. Determine the equation of the function graphed on the right.



- A.  $y = f(-2(x - 1))$
- B.  $y = f(-2(x + 1))$
- C.  $y = f\left(-\frac{1}{2}(x - 1)\right)$
- D.  $y = f\left(-\frac{1}{2}(x + 1)\right)$

22. The terminal arm of angle  $\theta$  in standard position intersects the unit circle at the point  $(m, n)$ . Which expression represents  $2 \csc \theta$

- A.  $\frac{2}{n}$
- B.  $\frac{2}{m}$
- C.  $2n$
- D.  $2m$

23. Determine the smallest positive solution of  $\sin x = \log x$ .
- A. 0.40
  - B. 1.93
  - C. 2.70
  - D. 3.71
24. In a circle, an arc of length 18 cm contains a central angle of  $240^\circ$ . Determine the radius of the circle.
- A. 24 cm
  - B. 36 cm
  - C. 4.30 cm
  - D. 75.36 cm
25. Determine the restriction(s) for the expression  $\frac{\csc x + 1}{\cot x}$
- A.  $\cos x \neq 0$
  - B.  $\sin x \neq 0$
  - C.  $\sin x \neq 0, \cos x \neq 0$
  - D.  $\sin x \neq 0, \cos x \neq 0, \csc x \neq -1$

26. Determine the sum of the infinite geometric series  $\frac{9}{25} + \frac{3}{5} + 1 + \dots$
- A.  $\frac{3}{5}$
  - B.  $-\frac{27}{50}$
  - C.  $\frac{27}{50}$
  - D. No finite sum
27. Determine the number of terms in the geometric sequence  $5, -10, 20, \dots, -163,840$
- A. 13
  - B. 14
  - C. 15
  - D. 16
28. The sum of an infinite geometric series is 12. If the common ratio is  $-\frac{1}{4}$ , determine the first term.
- A. 15
  - B. 16
  - C. 6
  - D. 5

29. Evaluate:  $\sum_1^4 \sin \frac{k\pi}{4}$

- A.  $1 + \sqrt{2}$
- B.  $-1$
- C.  $1$
- D.  $0$

30. Determine the value of the common ratio for the geometric sequence  $\sqrt{x}, 9, 3\sqrt{x}$ .

- A.  $3\sqrt{3}$
- B.  $\sqrt{3}$
- C.  $\frac{1}{\sqrt{3}}$
- D.  $3$

31. A new well produces 50 000L of water in the first month. If the volume of water pumped decreased by 8% each month, determine the total volume of water, in litres, that will be pumped from the well before it runs dry.

- A. 500 000
- B. 100 000
- C. 250 000
- D. 625 000

32. Which expression is equivalent to  $\log_3 15$

- A.  $\log 5$
- B.  $\log 45$
- C.  $5$
- D.  $\frac{\log 15}{\log 3}$

33. Solve  $\log_3(x+3) - \log_3(x-4) = 2$
- A.  $\frac{27}{5}$
  - B.  $\frac{39}{8}$
  - C. -3, 4
  - D. No solution
34. The inverse of  $y = \log x$  is:
- A.  $y = x$
  - B.  $y = 10x$
  - C.  $y = 10^x$
  - D.  $y = x^{10}$
35. Determine the number of different arrangements of all the letters in the word YTTUTORING.
- A. 604 800
  - B. 6 652 800
  - C. 3 628 800
  - D. 39 916 800
36. In BC, a license plate consists of 3 digits followed by 3 letters, **or 3 letters followed by 3 digits**. All digits 0 through 9 can be used. The letters I, O, Q cannot be used. If repetitions of letters and digits are allowed, determine the total number of possible license plates (e.g., 123 ABC, or ABB 007)
- A. 8 869 743
  - B. 17 739 486
  - C. 12 167 000
  - D. 24 334 000
37. In a standard deck of 52 cards, how many different 5-card hands are there that contain at most one face cards?
- A. 1 096 680
  - B. 1 754 688
  - C. 1 069 263
  - D. 1 645 020

38. In the expansion of  $(\frac{2}{3}a + \frac{3}{2}b)^6$ , determine of the coefficient of the middle term.
- A. 1  
B. 20  
C.  $\frac{80}{9}$   
D. No middle term, cannot be determined
39. YT Tutoring Centre has 4 different English text books, 5 different Math text books, and 3 **same** Physics text books to be displayed in a line on the book shelf. If the books with the same subject must put together, determine how many different arrangements are possible.
- A. 2 880  
B. 8 640  
C. 17 280  
D. 51 840
40. Two NBA players, Kobe Bryant and Yao Ming, each shoot a Free Throw shot at a score. The probability that Kobe scores a Free Throw shot is 85%. The probability that Yao scores a Free Throw shot is 90%. Assuming independence, what is the probability that none of them will score a goal?
- A. 0.765  
B. 0.985  
C. 0.22  
D. 0.015
41. A survey of YT Tutoring Center found the following:
- 10% of the students score  $\geq 95\%$  on their Math Provincial exam.
  - 60% of the students score  $\geq 90\%$  on their Math Provincial exam.
  - Only 10% of the students score  $< 86\%$  on their Math Provincial exam

What's the probability that a YT student will score  $\geq 86\%$  on their Math Provincial exam

- A. 90%  
B. 95%  
C. 30%  
D. Cannot be determined.



42. It is known that 2% of the population has a certain disease. A test for this disease is 95% accurate. This means that the outcome of the test is correct 95% of the time. What is the probability that a randomly selected person tests negative?
- A. 0.068
  - B. 0.053
  - C. 0.950
  - D. 0.932
43. There are 10 cars in a race. A particular bet requires a customer to choose the first three cars in the correct finishing order. If all 10 cars have an equal chance of finishing in any position, determine the probability that a single bet wins.
- A. 0.0014
  - B. 0.0083
  - C. 0.125
  - D. 0.3
44. A biased coin is designed so that it comes up heads 70% of the time. If this coin is tossed 10 times, determine the probability of obtaining between 4 and 6 heads inclusive.
- A. 0.0368
  - B. 0.0340
  - C. 0.2001
  - D. 0.3398

This is the end of multiple-choice section.  
Answer the remaining questions directly in the Response Booklet.

## PART B: WRITTEN RESPONSE

Value: 24 marks

Suggested Time: 30 minutes

**Instruction:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the graphing calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem it is important to sketch the graph, showing its general shape and indicating the appropriate values. If the statistical features of the calculator are used, it is important to show the function with the substitution of the relevant numbers. For example: In part of the solution it is acceptable to show `binomcdf (20,0.5,10)` or the equivalent syntax for the calculator used.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

**Full marks will NOT be given for the final answer only.**

1. How long will it take a certain type of bacteria to grow from 100 to 300 if the bacteria doubles in 23 weeks. (Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.) (5 marks)

2. Solve  $4 \sin^2 x \tan x - \tan x = 0$  (Answer as in exact value with General Solutions) (5 marks)

3. A sinusoidal curve has a minimum point at  $(-\frac{\pi}{2}, -5)$  and the closest maximum point to the right is  $(\frac{2\pi}{3}, 3)$ . Determine a sine equation for this curve. **(4 marks)**

4. It is known that 5% of the population has a certain disease. A test for this disease is 90% accurate. This means that the outcome of the test is correct 90% of the time. A positive test result claims that a person has the disease.

a. ) Determine the probability that a randomly selected person will test positive for this disease.  
(Answer accurate to at least 4 decimal places.) **(3 marks)**

b.) Given that a randomly selected person test positive, what is the probably that this person actually have the disease?

(Answer accurate to at least 4 decimal places.) **(2 marks)**

6. Prove the identities:

(5 marks)

$$\cot x(\sec^2 x - 1) = \frac{2 \cos x - 2 \cos^3 x}{\sin 2x \cos x}$$

LEFT SIDE

RIGHT SIDE