



First Name

Last Name

YT Tutoring Center

Principles of Mathematics 12

Sample Exam 2005

GENERAL INSTRUCTIONS

1. Aside from an approved calculator, electronic devices, including dictionaries and pagers, are **not** permitted in the examination room.
2. All multiple-choice answers must be entered on the Response Forms using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
3. For each of the written-response questions, write your answer in the space provided in this booklet. Rough-work space has been incorporated into the space allowed for answering each written-response question. You may not need all of the space provided to answer each question.
4. Ensure that you use language and content appropriate to the purpose and audience of this examination. Failure to comply may result in your paper being awarded a zero.
5. This examination is designed to be completed in **two hours**. *Students may, however, take up to 30 minutes of additional time to finish.*

PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

	Value	Suggested Time	Allowable Time
1. This examination consists of two parts:			
PART A:			
Section I: 16 multiple-choice questions	24 marks	35 minutes	45 minutes
Note: No calculator may be used for the first 45 minutes of the examination.			
Section II: 28 multiple-choice questions (some of which require the use of a calculator)	42 marks	55 minutes	} 105 minutes
PART B: 5 written-response topics covered by 8 questions	24 marks	30 minutes	
Total: 90 marks 120 minutes 150 minutes			

2. **After 45 minutes, the blue Response Form (Section I) will be collected. When all blue Response Forms are handed in, you will be permitted to use your calculator.**

During the first 45 minutes, you may proceed to other questions on the examination, many of which do not require the use of a calculator. Once the blue Response Forms have been handed in, you will **not** be able to go back to any of the first 16 questions; therefore, ensure you have checked your answers before proceeding to the rest of the examination.

3. The last **four** pages inside the back cover contain **A Summary of Basic Identities and Formulae, The Standard Normal Distribution Table, Rough Work for Graphing,** and **Rough Work for Multiple Choice.**
4. When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

PART A: MULTIPLE CHOICE (non-calculator)

SECTION I

Suggested Time: 35 minutes

Value: 24 marks

Allowable Time: 45 minutes

INSTRUCTIONS: No calculator may be used for this section of the examination. For each question, select the **best** answer and record your choice on the blue Response Form provided. Using an HB pencil, completely fill in the circle on the blue Response Form that has the letter corresponding to your answer.

1. Evaluate $\sum_{k=3}^5 \log_k k^2$
 - A. 1
 - B. 2
 - C. 6
 - D. 8

2. Simplify $\log_2 4^x$
 - A. x
 - B. 2x
 - C. 2^x
 - D. x^2

3. Given $\log_a 2 = x$ and $(\log_a 8)(a^{\log_a x}) = 12$, solve for a.
 - A. 2
 - B. ± 2
 - C. $\sqrt{2}$
 - D. $\pm\sqrt{2}$

4. Determine the period of $y = \sin \frac{2\pi}{3}(x - 6)$.
 - A. 3
 - B. 6
 - C. 3π
 - D. 4π

5. Solve: $1 + 2 \sin x = 0$, $0 \leq x < 2\pi$.

- A. $-\frac{\pi}{6}$
- B. $\frac{2\pi}{3}, \frac{5\pi}{3}$
- C. $\frac{2\pi}{3}, -\frac{\pi}{3}$
- D. $\frac{7\pi}{6}, \frac{11\pi}{6}$

6. Identify the conic that is described by $x^2 + 6y^2 - 18 - 45 = 0$

- A. Circle
- B. Ellipse
- C. Parabola
- D. Hyperbola

7. If m and n are positive integers, determine the radius of the circle $mx^2 + my^2 - n = 0$

- A. $\sqrt{\frac{n}{m}}$
- B. $\sqrt{\frac{m}{n}}$
- C. $\frac{n}{m}$
- D. $\frac{m}{n}$

8. How is the graph $3y = \sqrt{x}$ related to the graph $y = \sqrt{x}$?

- A. $y = \sqrt{x}$ has been vertically translated 3 units up.
- B. $y = \sqrt{x}$ has been expanded vertically by a factor of 3.
- C. $y = \sqrt{x}$ has been compressed vertically by a factor of $1/3$.
- D. $y = \sqrt{x}$ has been compressed horizontally by a factor of $1/3$.

9. How many terms are there in the series defined by $\sum_{28}^{55} (2x + 5)$
- A. 27
 B. 28
 C. 29
 D. 30
10. Determine the common ratio of the geometric sequence $\log x^2, \log x^6, \log x^{18}$, where $x > 0$
- A. x
 B. x^3
 C. $\log x^3$
 D. 3
11. Solve for $x : a^{x+3} = b^{x-2}$
- A. $\frac{2 \log b - 3 \log a}{\log b - \log a}$
 B. $\frac{2 \log b - 3 \log a}{\log a - \log b}$
 C. $\frac{2 \log b + 3 \log a}{\log b - \log a}$
 D. $\frac{2 \log b + 3 \log a}{\log a - \log b}$
12. Given a standard normal curve, determine the approximate value of $P(0 < Z < 1)$
- A. 34%
 B. 68%
 C. 95%
 D. 99.7%
13. Simplify the following expression without using the factorial symbol $\frac{(n+2)!(n-1)!}{((n+1)!)^2}$.
- A. $\frac{1}{n+1}$
 B. $\frac{(n+2)}{(n+1)(n)^2}$
 C. $\frac{(n+2)}{(n+1)(n)}$
 D. $\frac{(n-1)}{(n+1)(n)}$

14. Solve for n: ${}_nP_2 = 42$
- A. 2
 - B. 6
 - C. 7
 - D. 42
15. If the graph of $x^2 + y^2 = 1$ is vertically compressed by a factor of $1/5$, and horizontal expanded by a factor of 2, determine an equation for the new graph.
- A. $\frac{x^2}{2} + 5y^2 = 1$
 - B. $\frac{x^2}{4} + 25y^2 = 1$
 - C. $2x^2 + \frac{y^2}{5} = 1$
 - D. $4x^2 + \frac{y^2}{25} = 1$
16. Solve: $\log_5(2x+1) = 1 - \log_5(X+2)$
- A. $1/2$ or -3
 - B. -3
 - C. $1/2$
 - D. No solution

PART A: MULTIPLE CHOICE
SECTION II

Value: 42 marks

Suggested Time: 55 minutes

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the blue Response Form provided. Using an HB pencil, completely fill in the circle on the blue Response Form that has the letter corresponding to your answer.

17. Determine the minimum value of the function $y = -3 \sin 3x - 5$

- A. -2
- B. 8
- C. -8
- D. 2

18. Determine the phase shift of the function: $f(x) = \sin\left(\frac{1}{4}x - \pi\right)$

- A. π to the right
- B. π to the left
- C. 4π to the right
- D. $\frac{1}{4}\pi$ to the right

19. If $A = B + 90^\circ$, simplify $\sin A \cos B - \cos A \sin B$

- A. 0
- B. 1
- C. $\sin 2B$
- D. $\sin(2B + 90^\circ)$

20. Determine the exact value of $\sec\left(-\frac{3\pi}{4}\right)$

- A. $-\frac{2}{\sqrt{2}}$
- B. $-\frac{\sqrt{2}}{2}$
- C. $\frac{\sqrt{2}}{2}$
- D. $\frac{2}{\sqrt{2}}$

21. Determine the equation of the asymptote of $y = 4 \log_2(x+2) - 3$
- $x=-2$
 - $x=2$
 - $y=-3$
 - $y=3$
22. Simplify: $3^{3\log_3 3x}$
- $3x$
 - $9x^2$
 - $9x^3$
 - $27x^3$
23. Determine the value of $\log_x ab^3$ if $\log_x a = 4$ and $\log_x b = 3$.
- 12
 - 13
 - 21
 - 81
24. An earthquake in Vancouver measured 5.2 on the Richter scale and an earthquake in San Francisco measured 7.1. How many times as intense was the earthquake in San Francisco compared to the earthquake in Vancouver?
- 1.90
 - 1.37
 - 79.43
 - 36.92
25. What equation represents the graph of $y = g(x)$ after it is reflected in the line $y=x$?
- $y=g(-x)$
 - $y=-g(x)$
 - $y= 1/g(x)$
 - $x = g(y)$
26. If the range of $y=f(x)$ is $-2 \leq y \leq 3$, what's the range of $y = \frac{1}{f(x)}$?
- $-2 \leq y \leq 3$
 - $-2 \leq y \leq 1/3$
 - $-1/2 \leq y \leq 1/3, y \neq 0$

D. $y \leq -\frac{1}{2}, y \geq \frac{1}{3}$

27. Determine the equation of the inverse of $f(x) = \frac{x}{x-1}$

A. $f^{-1}(x) = \frac{x}{x-1}$

B. $f^{-1}(x) = \frac{x}{x+1}$

C. $f^{-1}(x) = \frac{x}{1-x}$

D. $f^{-1}(x) = \frac{-x}{1-x}$

28. The zeros of the function $y = f(x)$ are -2, 0 and 3. Determine the zeros of the new function $y = -f(x+1)$?

A. -3, -1, 2

B. -1, 1, 4

C. 2, 0, -3

D. 1, -2, -4

29. The sum of an infinite geometric series is 12. If the common ratio is $-1/2$, determine the first term.

A. 18

B. 12

C. 24

D. 36

30. Evaluate: $\sum_1^4 \sin \frac{k\pi}{4}$

A. $1 + \sqrt{2}$

B. -1

C. 1

D. 0

31. Determine the sum of the first 10 terms of the geometric series defined by $1/2 - 2 + 8 - 32 + \dots$

A. 104857.5

B. -104857.5

C. 511.5

D. -511.5

32. A new well produces 50 000L of water in the first month. If the volume of water pumped decreased by 8% each month, determine the total volume of water, in litres, that will be pumped from the well before it runs dry.
- A. 500 000
B. 100 000
C. 250 000
D. 625 000
33. Determine the number of terms in the geometric sequence: $1/64$, $1/16$, $1/4$, ... , 4096.
- A. 9
B. 10
C. 11
D. 12
34. Determine the length of the major axis of the ellipse $5x^2+8y^2=40$.
- A. 8
B. 16
C. $2\sqrt{2}$
D. $4\sqrt{2}$
35. Given the hyperbola $\frac{(x-2)^2}{25} - \frac{(y+3)^2}{49} = 1$, determine an equation of one of its asymptotes.
- A. $y = \frac{7}{5}x + \frac{1}{5}$
B. $y = \frac{7}{5}x - \frac{29}{5}$
C. $y = \frac{49}{25}x - \frac{29}{5}$
D. $y = \frac{49}{25}x + \frac{1}{5}$

36. If $Ax^2 + By^2 = 1$ is an ellipse with its minor axis on the y-axis, determine the length of the major axis.
- A. $2A^2$
 B. $2\sqrt{A}$
 C. $\frac{2}{A^2}$
 D. $\frac{2}{\sqrt{A}}$
37. Determine an equation of the hyperbola that has centre (4, 6), a vertices (10, 6) and an asymptote $x-2y+8=0$.
- A. $\frac{(x+4)^2}{36} - \frac{(y+6)^2}{9} = 1$
 B. $\frac{(x+4)^2}{6} - \frac{(y+6)^2}{9} = 1$
 C. $\frac{(x-4)^2}{6} - \frac{(y-6)^2}{9} = 1$
 D. $\frac{(x-4)^2}{36} - \frac{(y-6)^2}{9} = 1$
38. It is known that 2% of the population has a certain disease. A test for this disease is 95% accurate. This means that the outcome of the test is correct 95% of the time. What is the probability that a randomly selected person tests negative?
- A. 0.068
 B. 0.053
 C. 0.950
 D. 0.932
39. A biased coin is designed so that it comes up heads 70% of the time. If this coin is tossed 10 times, determine the probability of obtaining between 4 and 6 heads inclusive.
- A. 0.0368
 B. 0.0340
 C. 0.2001
 D. 0.3398

40. There are 10 cars in race. A particular bet requires a customer to choose the first three cars in the correct finishing order. If all 10 cars have an equal chance of finishing in any position, determine the probability that a single bet wins.
- A. 0.0014
 - B. 0.0083
 - C. 0.125
 - D. 0.3
41. It is estimated that 30% of teenagers smoke at least 10 cigarettes daily. Use the normal approximation to the binomial distribution to determine the probability that between 50 and 60 (inclusive) teenagers from 200 randomly chosen teenagers smoke at least 10 cigarettes daily.
- A. 0.4386
 - B. 0.4782
 - C. 0.5348
 - D. 0.4842
42. The diameters of orange are normally distributed with a mean diameter of 9.0cm and a standard deviation of 0.8cm. What is the largest diameter that would be less than 85% of the oranges?
- A. 9.83cm
 - B. 8.17cm
 - C. 8.20cm
 - D. 9.80cm
43. A population of scores is normally distributed with a mean of 23.0 and a standard deviation of 15.1. If 45% scores are higher than a particular score x , calculate the value of x .
- A. 21.10
 - B. 24.90
 - C. 28.82
 - D. 33.35
44. A fair **ten**-side die is rolled n times. If the standard deviation of the number of times a 6 comes up is 9.49, determine the value of n .
- A. 1001
 - B. 999
 - C. 980
 - D. 1100

This is the end of multiple-choice section.

PART B: WRITTEN RESPONSE

Value: 24 marks

Suggested Time: 30 minutes

Instruction: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the graphing calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem it is important to sketch the graph, showing its general shape and indicating the appropriate values. If the statistical features of the calculator are used, it is important to show the function with the substitution of the relevant numbers. For example: I part of the solution it is acceptable to show `normalcdf (10, 40, 50, 20)` or the equivalent syntax for the calculator used.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

1. The population of a nest of bees can multiple threefold (triple) in 10 weeks. If the population is now 13000, how many weeks will it take for the population to reach 50000 bees? (Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.) **(4 marks)**

2. The circle with equation $x^2 + 8x + y^2 + 4y = 0$ is translated 2 units to the right to form a new circle. Determine the equation of the new circle and change to standard form. (4 marks)

3. A sinusoidal curve has a minimum point at $(-\frac{\pi}{2}, -5)$ and the closest maximum point to the right is $(\frac{\pi}{3}, 3)$. Determine an equation of this curve. **(4 marks)**

4. a.) Picking 5 cards from a deck of card, what is the probability to obtain exactly 3 Kings'?
(Answer accurate to at least 4 decimal places.) **(2 marks)**

b.) Picking 5 cards from a deck of card, what's the probability to obtain at least 3 Kings'?
(Answer accurate to at least 4 decimal places.) **(2 marks)**

5. In BC, the probability that a student can go to any post-secondary institution is 0.60. If 300 students are randomly selected, determine the probability that between 200 and 202 students inclusive can attend any post-secondary institution by using the following methods.

a. Use the binomial distribution to obtain this probability
(Answer accurate to at least 4 decimal places.)

(2 marks)

b. Use the normal approximation to the binomial distribution to obtain an estimate of this probability.
(Answer accurate to at least 4 decimal places.)

(2 marks)

6. Prove the identities:

(4 marks)

$$\cot^2 \theta \sin 2\theta + \sin 2\theta = 2 \cot \theta$$