First Name

Last Name

YT Tutoring Center

Principles of Mathematics 12

Sample Exam 2004

PART A : MULTIPLE CHOICE

Value: 63 marks	Suggested Time: 75 minutes

1. Convert 120 degree to radians.
   A. 2.09
   B. 0.17
   C. 4.19
   D. 1.05

2. Determine the maximum value of the function $y = -3\sin\theta - 7$
   A. -4
   B. -10
   C. 4
   D. 10

3. Determine the exact value of value of $\cot(\pi/4)$
   A. 1/4
   B. 1
   C. 0
   D. undefined

4. Solve: $\tan\theta = -1.25,\ 0 < \theta < 2\pi$
   A. 0.90, 4.04
   B. 2.25, 5.39
   C. 2..25, 6.15
   D. 3.01, 6.15

5. Simplify 10 sin 20x
   A. 10 sin20x cos20x
   B. 20 sin20x cos20x
   C. 20 sin10x cos10x
   D. 10 sin10x cos10x

6. The smallest positive solution of tan bx = 4 is x = 0.5. Determine the general solution of tan bx = 4.
   A. 0.5 + 2Πn, n is an integer
   B. 0.5 + 2bΠn, n is an integer
   C. 0.5 + Πb/n, n is an integer
   D. 0.5 + 2nΠ/b, n is an integer

7. At a seaport, the water has a maximum depth of 20 m at 5:00am. After this maximum depth, the first minimum depth of 4 m occurs at 10:00am. Assume that the relation between the depth, h meters, and the time, t hours, s a sinusoidal function. Determine an equation for h at anytime t.
   A. $12 \cos \ 2\pi \frac{(t-5)}{5} + 8$
   B. $12 \cos \ 2\pi \frac{(t-5)}{10} + 8$
   C. $8 \cos \ 2\pi \frac{(t-5)}{5} + 12$
   D. $8 \cos \ 2\pi \frac{(t-5)}{10} + 12$

8. Determine the ratio of the geometric sequence $2x, 4x^4, \ldots$, where $x \neq 0$.
   A. $2x$
   B. x^3
   C. $2x^3$
   D. $2x^4$

9. How many terms are there in the series defined by $\sum_{28}^{55}(2x+5)$
   A. 27
   B. 28
   C. 29
   D. 30

10. Determine the 10^{th} term of the geometric sequence: -10, 25, -62.5, …
    A. -38146.97
    B. -15258.79
    C. -95367.43
    D. 38146.97

11. In a geometric sequence, $t_2 = 3$, $t_6=243/16$. Determine the common ratio, r.
    A. $\frac{3}{2}$
    B. $-\frac{3}{2}$
    C. $\pm\frac{3}{2}$
    D. 4/3

12. Determine the common ratio of the geometric sequence $\log x^2$, $\log x^6$, $\log x^{18}$, where $x>0$
    A. x
    B. x^3
    C. $\log x^3$
    D. 3

13. Consider the geometric sequence 1, (a+b), $(a+b)^2$, … Which term of this geometric sequence, when expanded, contains the expression $70a^4b^4$.
    A. 6^{th} term
    B. 7^{th} term
    C. 8^{th} term
    D. 9^{th} term

14. Consider the geometric sequence $(a+b)$, $(a+b)^2$, … Which term of this geometric sequence, when expanded, contains the expression $56a^5b^3$.
    A. 6^{th} term
    B. 7^{th} term
    C. 8^{th} term
    D. 9^{th} term

15. Determine the domain of $y= - \log_a(-x)$
    A. $x>0$
    B. $x<0$
    C. $x \geq 0$
    D. $x \leq 0$

16. Which expression is equivalent to $\log \frac{1000x^2}{y^2z^3}$
    A. $\log 3 + 2 \log x - 2 \log y + 3 \log z$
    B. $\log 1000 + 2 \log x - 2 \log y + 3 \log z$
    C. $1000 + 2 \log x - 2 \log y - 3 \log z$
    D. $3 + 2 \log x - 2 \log y - 3 \log z$

17. If the point (3, 5) is on the graph of $y = a^x$, what point must be on the graph of $y = - \log_a x$?
    A. (3, 5)
    B. (5, -3)
    C. (-3, -5)
    D. (-5, -3)

18. Solve $(1/81)^{3x-2} = 27^{x+6}$
    A. -2/3
    B. -3/2
    C. 2/3
    D. 3/2

19. Determine the magnitude of an earthquake that is 16 times as intense as an earthquake of magnitude 5.5 on the Richter scale.
    A. 7.0
    B. 6.7
    C. 6.0
    D. 5.5

20. Solve: $\log_5(2x+1) = 1 - \log_5(X+2)$
    A. 1/2 or -3
    B. -3
    C. 1/2
    D. No solution

21. Which conic is described by the equation $4x^2 - 4y^2 - x + y = 0$?
    A. Circle
    B. Ellipse
    C. Parabola
    D. Hyperbola

22. Which equation represents the graph of y = f(x) after it is translated 5 units right?
    A. y = g(x) + 5
    B. y = g(x) -5
    C. y = g(x+5)
    D. y = g(x-5)

23. Determine the length of the minor axis of the ellipse $6x^2 + 8y^2 - 48 = 0$?
    A. 6
    B. 8
    C. $\sqrt{6}$
    D. $2\sqrt{6}$

24. Solve for x : $a^{x+3} = b^{x-2}$

A. $\frac{2 \log b - 3 \log a}{\log b - \log a}$

B. $\frac{2 \log b - 3 \log a}{\log a - \log b}$

C. $\frac{2 \log b + 3 \log a}{\log b - \log a}$

D. $\frac{2 \log b + 3 \log a}{\log a - \log b}$

25. Give the vertices of $y^2 - (x-2)^2 = 4$

A. (2, 2) , (2, -2)
B. (2, 4) , (2, -4)
C. (4, 2), (4, -2)
D. (-2, 2), (2, 2)

26. If the point (2, 4) is on the graph of y = f(x), what point must be on the graph of $y = 4 \left[\frac{1}{f(x)} \right]$

A. (2, 1)
B. (2, 1/16)
C. (2, 4)
D. (8, 1)

27. Determine an equation that will cause the graph of y = f(x) to expand vertically by a factor of 3 and reflect in the x – axis.

A. $y = -3f(x)$

B. $y = -\frac{1}{3} f(x)$

C. $y = \frac{1}{3} f(-x)$

D. $y = 3f(-x)$

28. How many different ways are there to arrange the books on the bookshelf if there're 3 same English books, 2 different chemistry books, and 3 same physics books?
    A. 18
    B. 72
    C. 1120
    D. 40320

29. A 7-card hand is dealt from a standard deck of 52 cards. What is the probability that the hand will contain 4 clubs and 3 red cards?
    A. 0.0320
    B. 0.2905
    C. 0.0139
    D. 0.0246

30. In a recent survey of grade 12 students, it was found that 50% need a mathematics tutor and 50% need a chemistry tutor as well. If 25% of students need both mathematics and chemistry tutor, what percent of students don't need a tutor on those two subjects?
    A. 25%
    B. 50%
    C. 70%
    D. not enough information

31. A bag contains 4 white balls and 4 black balls. Two balls are drawn one at a time with replacement. What is the probability that both balls are the same color?
    A. 1/2
    B. 1/4
    C. 1/8
    D. 24/56

32. Determine the 4^{th} term of $(2x - 3y)^5$.
    A. $162xy^4$
    B. $810xy^4$
    C. $162x^2y^3$
    D. $-1080x^2y^3$

33. Calculate the probability of tossing the coin 20 times and getting exactly 9 heads.
    A. 0.60
    B. 0.40
    C. 0.26
    D. 0.16

34. During a summer season, a golfer plays 30 rounds with scores as shown in the table below. Calculate the mean and the standard deviation of the population of scores.

Score	Number of rounds
50	3
51	6
52	9
53	12

    A. 51.96, 1.06
    B. 51.96, 1.04
    C. 52.00, 1.00
    D. 52.00, 1.02

35. For a standard normal distribution, determine the probability that a z-score is greater than 1.0
    A. 0.3200
    B. 0.6800
    C. 0.3137
    D. 0.1587

36. Given a standard normal curve, determine the approximate value of $P(0 < Z < 1)$
    A. 34%
    B. 68%
    C. 95%
    D. 99.7%

37. A population of scores is normally distributed with a mean of 55.5, and a standard deviation of 10.6. If 40% of the scores are higher than a particular score x, calculate the value of x.
    A. 58.19
    B. 52.81
    C. 24.66
    D. 21.33

38. The heights of basketball players at BC Provincial Tournaments are normally distributed with a mean of 175.8cm and a standard deviation of 16.5cm. What percent of players at these tournaments have heights between 170cm to 190cm?
    A. 42%
    B. 44%
    C. 46%
    D. 48%

39. A fair six-sided die is rolled n times. If the standard deviation of the number of times a 4 comes up is 5, determine the value of n.
    A. 180
    B. 157
    C. 156
    D. 120

40. Determine the mean if the z-score is 1.4, X value is 5.2 and the standard deviation is 2.2
    A. 12.12
    B. 10.12
    C. 8.12
    D. 2.12

41. The weights of a population are normally distributed. Approximately what percent of the population has a weight that is with 3 standard deviations of the mean?
    A. 68%
    B. 95%
    C. 99.7%
    D. 100%

42. If Z has a standard normal distribution with P (a<Z<b) = 0.1, where -3<a<b<3, which of the following statements must be true?
    A. P(Z<a) > 0.1
    B. P(Z>a) > 0.1
    C. P(Z<b) < 0.1
    D. P(Z>b) < 0.1

This is the end of multiple-choice section.

PART B: WRITTEN RESPONSE

Value: 27 marks	Suggested Time: 45 minutes

Instruction: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the graphing calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem it is important to sketch the graph, showing its general shape and indicating the appropriate values. If the statistical features of the calculator are used, it is important to show the function with the substitution of the relevant numbers. For example: I part of the solution it is acceptable to show normalcdf (10, 40, 50, 20) or the equivalent syntax for the calculator used.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

1. a.) A theatre company of 14 actors consists of 8 men and 6 women. How many different ways are there to choose from the theatre company a group of 8 with exactly 3 men? **(2 marks)**

b) A theatre company of 14 actors consists of 8 men and 6 women. How many different ways are there to choose from the theatre company a group of 8 with at least 4 women? **(2 marks)**

2. Solve algebraically, giving exact values, where $0 \leq x < 2\pi$. **(4 marks)**

$$2 \tan x \cos x - \tan x = 0$$

3. Solve algebraically: $\log_2 x + \log_2(x-7) = 3$ **(5 marks)**

4. a.) Change to standard form: $4y^2 + 16y - 9x^2 + 18x - 29 = 0$

(4 marks)

b.) graph $4y^2 + 16y - 9x^2 + 18x - 29 = 0$ **(2 marks)**

5. In a large city in BC the probability that a car has air conditioning is 0.72. If 300 cars are randomly selected, determine the probability that between 210 to 212 cars inclusive have air conditioning by using the following methods.

   a. Use the binomial distribution to obtain this probability. (Answer accurate to at least 4 decimal places.) **(2 marks)**

   b. Use the normal approximation to the binomial distribution to obtain an estimate of this probability. (Answer accurate to at least 4 decimal places.) **(2 marks)**

6. Prove the identity: **(4 marks)**

$$\cot \theta - \cos 2\theta \cot \theta = \sin 2\theta$$

END OF EXAMINATION

A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Reciprocal and Quotient Identities:

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Addition Identities:

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \qquad \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

Double-Angle Identities:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \qquad \sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

Formulae:

$$t_n = ar^{n-1} \qquad S_n = \frac{a(1-r^n)}{1-r} \qquad S_n = \frac{a-r\ell}{1-r} \qquad S = \frac{a}{1-r} \qquad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Probability and Statistics:

$${}_nP_r = \frac{n!}{(n-r)!} \qquad {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} \qquad t_{k+1} = {}_nC_k\, a^{n-k}\, b^k$$

$$P(\overline{A}) = 1 - P(A) \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} \qquad P(A \text{ and } B) = P(A) \times P(B \mid A)$$

$$P(x) = {}_nC_x\, p^x q^{n-x} \qquad (q = 1-p)$$

$$\mu = \frac{\sum x_i}{n} \qquad \sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{n}} \qquad \mu = np \qquad \sigma = \sqrt{npq} \qquad z = \frac{x-\mu}{\sigma}$$

Note: Graphing calculators will contain many of these formulae as pre-programmed functions.

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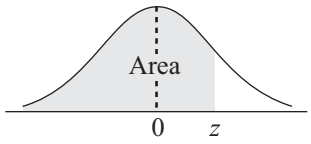
THE STANDARD NORMAL DISTRIBUTION TABLE



$$F_z(z) = P[Z < z]$$

z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
–3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
–3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
–3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
–3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
–3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
–2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
–2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
–2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
–2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
–2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
–2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
–2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
–2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
–2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
–2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
–1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
–1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
–1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
–1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
–1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
–1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
–1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
–1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
–1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
–1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
–0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
–0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
–0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
–0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
–0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
–0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
–0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
–0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
–0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
–0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

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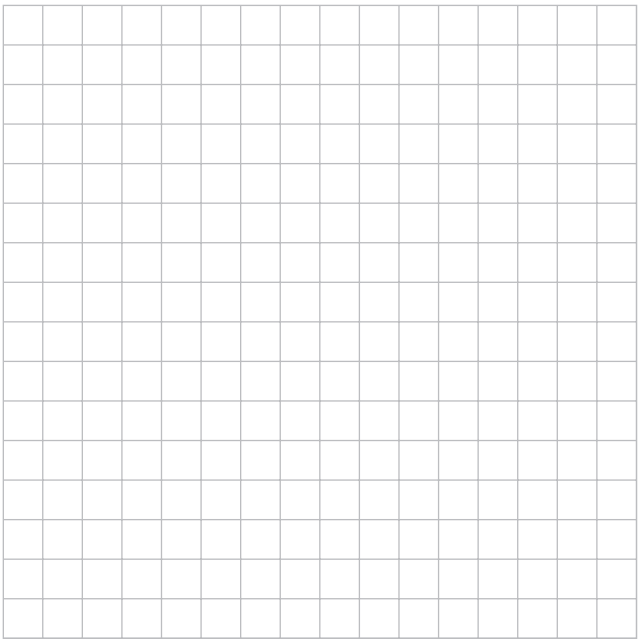
Exercise care when tearing along perforations.

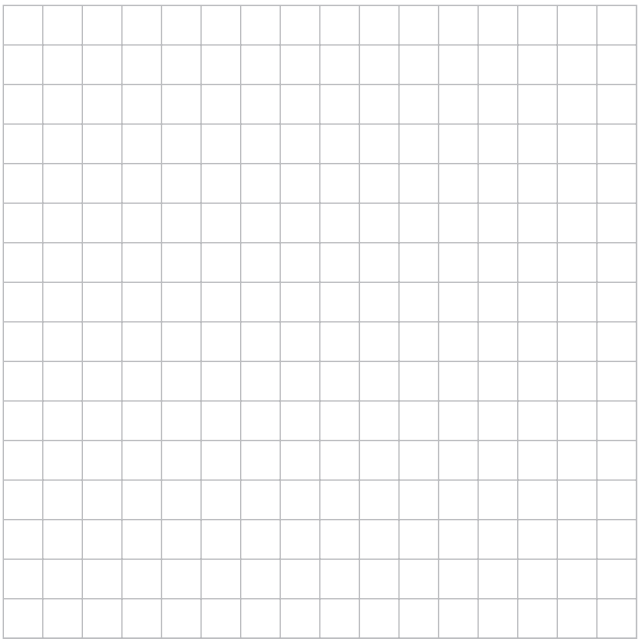
$$F_z(z) = P[Z < z]$$

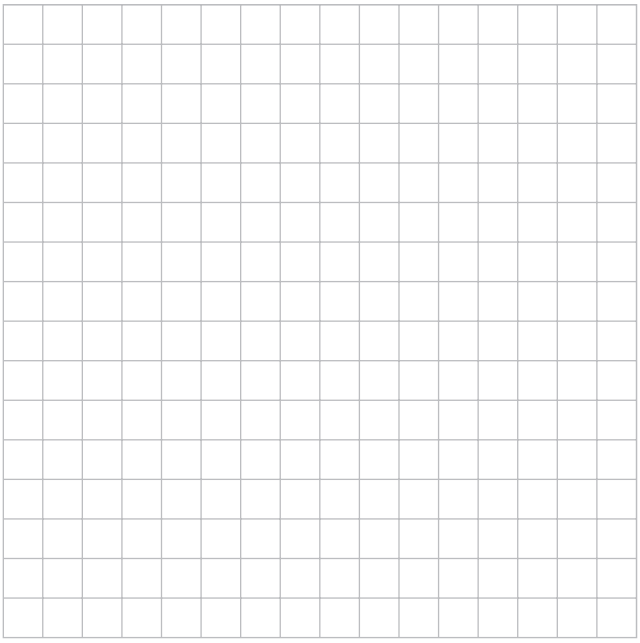
z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

ROUGH WORK FOR GRAPHING

(No marks will be given for work done on this page.)







You may detach this page for convenient reference.
Exercise care when tearing along perforations.